## Appendix 3

## The Effect of Water Markets on Irrigation Technology Adoption

# THE EFFECT OF WATER MARKETS ON IRRIGATION TECHNOLOGY ADOPTION

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by

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## I. INTRODUCTION

This paper examines a farm's decision to adopt modern irrigation technology with and without the availability of a water market. When there is no market, production is determined by the farm's stochastic water allocation $x_0$. The farm initially produces with traditional irrigation technology, but if its allocation drops to a threshold level $\underline{x}_0$, it may pay a sunk cost $I$ and invest in the modern technology. It is also possible, if the cost of investment is prohibitively high and/or if the gains from the modern technology are not great, that the farm will never invest. In these cases, the farm will continue to produce with the traditional technology until $x_0$ drops so low that it stops producing all together.

When there is a market, the farm can adjust its water supply in response to changes in the market price $p$. The farm initially produces with traditional technology, but if the market price rises to a threshold level $\overline{p}$, it may pay the investment cost $I$ and switch to the modern technology. Although the farm's profit with either the traditional or the modern technology depends on its allocation level $x_0$, the farm's technology adoption decision is independent of $x_0$, because the *difference* in profit between the two technologies is not a function of $x_0$. As in the no-market case, the farm may never invest in the modern technology if the net gains of adoption are too low. Eventually, the price may increase to a cutoff level at which the farm sells all of its water rather than producing with either technology.

Suppose threshold levels, $\underline{x}_0$ and $\overline{p}$, do exist at which it is profitable to invest in the modern technology. Whether the farm invests earlier with or without access to a water market depends on the time paths of $x_0$ and $p$. If $p$ rises to $\overline{p}$ before (after) $x_0$ falls to $\underline{x}_0$, the farm would adopt earlier (later) with market access than without market access. The correlation between $x_0$ and $p$ may depend on the relative seniority of the farm's water rights. If for example, the farm has very senior rights, $x_0$ may remain relatively constant while the market price increases over time. In this case, the farm might never adopt without market access; however, it might adopt if it has market access. If instead the farm has junior rights, and faces large cutbacks in its allocation relative to other water users, it might adopt earlier without market access.

The paper is organized as follows. Section II provides a brief review of related papers in the technology adoption literature. Section III develops the model of technology adoption in the non-market case, and section IV develops the model of adoption in the market case. Section V compares the technology adoption decision under the two systems, and section VI discusses the impact of including transaction costs in the analysis.





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## II. LITERATURE REVIEW

While there is an extensive literature on irrigation technology adoption, surprisingly few studies have analyzed the effect of water markets on technology adoption. The work which has examined the effect of water markets on technology adoption does not consider the impact of dynamics or uncertainty. The analysis of non-market versus market water allocation systems is very similar to the analysis of fixed environmental standards versus tradable emissions permit systems. A few studies exist which examine the effect of tradable permit systems on pollution abatement technology adoption; however, here again, the models do not address issues of dynamics and uncertainty. As argued in the introduction, the greatest value of switching to market-based systems may not be the value of the increase in allocative efficiency at any given time, but the value of the increase in dynamic efficiency associated with technology adoption and other long-term investments. To test this argument, we need a dynamic model of technology adoption which takes into account the effect water supply uncertainty and water price uncertainty on investment decisions.

Two seminal articles on irrigation technology adoption by Caswell *et al.* [1, 2] examine the effect of well depth and land quality on the choice of irrigation technology and the effect of a drainage effluent charge on the choice of technology. In both papers, a farmer chooses between two irrigation technologies, a traditional technology and a modern water-saving technology, by solving a two-part optimization problem. First, the farmer chooses the optimal input quantities with each technology and second, the farmer chooses the technology which generates the greatest profit, with the constraint that the profit is positive. Given that the models in Caswell are static, the two-part optimization takes place instantaneously. In contrast, the dynamic model in this paper assumes that the farm is currently producing with the traditional technology and it must decide when, if ever, to switch to the modern technology. While significantly different from the Caswell models, the model in this paper has adopted the same quadratic production function technology used in the Caswell papers. Using California cotton production data, Caswell (1986) finds that the quadratic production function provides more reasonable simulation results than a Cobb-Douglas production function [1]. Future versions of this paper may employ a more general production function.

Shah and Zilberman [3] analyze the effect on technology adoption of switching from non-market water allocation to water markets, using a static model with multiple technology choices. The non-market system they employ is a "queuing" system based on prior appropriation doctrine. Under their queuing system, senior rights holders have access to as much water as they want and junior rights holders have access to whatever water is left over. Transfers between senior- and junior-rights holders are prohibited. Since their model is static, there is no sense of how supplies to senior- and junior-rights holders change over time as supplies fluctuate. Shah and Zilberman contrast adoption behavior under a queuing system with a behavior under a water market, but rather than solving the decentralized market equilibrium directly, they assume that the necessary conditions for the First Fundamental Theorem hold, and solve it indirectly via the social planner's optimization problem. Shah and Zilberman conclude that a switch from queuing to a market system will increase the adoption of modern irrigation technology. However, their conclusion seems to rest on an analysis of the behavior of senior-rights holders. It does not address the possibility that junior-rights holders may have less incentive to adopt modern technology under a market system.

In this paper, the non-market allocation system is general enough to describe either a queuing system like Shah and Zilberman's or a system in which supplies are rationed proportionally among users. An individual farm's allocation fluctuates over time in response to shifts in aggregate supply and demand. A farm receives the same initial allocation regardless of whether there is a non-market or a market system, but with a market the farm has the ability to reduce or increase its supply by trading in the market. The comparison between the non-market and market systems is closely related to the comparison between a fixed pollution standard and a tradable emission permit system. Using a static graphical analysis, Malueg [4] demonstrates that a shift from a fixed standard to a tradable permit system may increase *or decrease* the incentive for firms to switch from a high-cost to a low-cost pollution abatement technology. In his model, if a firm is a seller (buyer) of permits with both the high-cost and the low-cost technology, the gains from adopting the low-cost technology are greater (less) under the tradable permit system. If a firm buys permits using the high-cost technology and sells permits using the low-cost technology, then the gains from adopting the low-cost technology may increase or decrease under the tradable permit system. The gains will increase (decrease) if the firm buys relatively fewer (more) permits with the high-cost technology than it sells with the low-cost technology. This paper uses Malueg's framework to compare the irrigation technology adoption decision under a non-market and a market system. The static graphical example of the adoption decision is used to motivate the intuition for the stochastic dynamic analysis.

The stochastic dynamic investment model developed in this paper uses option value techniques common in the finance literature and popularized in a recent book by Dixit and Pindyck [5]. Farms face significant uncertainty, regarding future water supplies and





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prices, which affects the value of investments in modern irrigation technology. The static models described above do not capture the effect of this dynamic uncertainty on a farm's investment strategy. Dynamic models which employ traditional cost-benefit analysis also do not adequately account for the effect of uncertainty. Traditional cost-benefit models of investment predict that a firm will invest when the expected present value of investment equals the cost of investment. However, Dixit and Pindyck show that when investment is characterized by uncertainty, irreversibility and the ability to wait for more information, firms should not invest until the expected present value of investment *exceeds* the cost of investment. This rule is more consistent with observed investment behavior. Firms require an expected return greater than the investment cost because when they invest, they give up the *option* to invest. The option to invest has a positive value because, by waiting, firms can obtain more information before committing to a sunk investment cost. Firms should wait to invest until the expected present value of investment equals the cost of investment *plus the value of the option to invest.*

In a basic option value model of investment, McDonald and Siegel [6] consider a firm's decision pay a sunk cost $I$ in return for a project whose value, $V$, is stochastic. In other models, the value of the project is an explicit function of an output price $P$, which is stochastic. The McDonald and Siegel model analyzes the decision to invest in a project which generates no returns until after the investment is made. Once the investment is made, the value of the project evolves according to a exogenous stochastic process. The model does not specify a production process which would allow the firm to vary its inputs in response to realized values. Other models, known as optimal stopping problems, examine the decision to disinvest. Before the disinvestment is made, the project produces a stochastic benefit (or cost) flow, and after the disinvestment, the flow is zero. This paper differs from these models in three respects. First, instead of the usual output uncertainty, this paper focuses on input uncertainty: the farm's initial water allocation $x_0$ and the price of water $p$ are stochastic. Second, this paper analyzes the decision of a farm to *switch production technologies.* Because the farm is already producing before the investment in new technology is made, the farm generates a positive income flow both before and after the investment. Hassett and Metcalf [7] and Herbelot [8] also employ the option value approach in technology switching models.

Hassett and Metcalf studied residential energy conservation investments and found that consumers' responses to investment tax credits were very low. If consumers were making their decisions based on net present value theory, they had to have been using extremely high discount rates. Hasset and Metcalf develop a model in which investments are irreversible and the price of heating fuels fluctuates randomly over time. Given the irreversibility and uncertainty of investments, the model predicts that individuals will wait until the investment return is significantly above the investment cost. The household investment data support their model. They simulate the effect of an investment tax credit and find that, if uncertainty is ignored, the effect of the tax credit is significant. However, when uncertainty is taken into account, the tax credit has very little impact.

Herbelot employs the option value approach in an analysis of electric utilities' efforts to comply with $SO_2$ emissions regulations. An electric utility can comply with the regulations by purchasing permits from other utilities, or by switching to a low-sulfur fuel or installing scrubbers. If the utility switches fuels it must pay a sunk cost to retrofit the plant, and if it installs scrubbers it must also pay a sunk capital cost. In addition, the price of emission allowances and the price premium on low-sulfur fuel fluctuate stochastically. Herbelot shows that the utility may choose to purchase emission allowances, even if the expected present value of compliance is higher with the allowances, because of the flexibility they provide. Even if the utility does not decide to switch fuels or install scrubbers, Herbelot argues that the utility's true compliance cost is lower because it has the *option* to switch fuels or install scrubbers.

Herbelot's approach most closely resembles the one employed in this paper. Herbelot does not, however, compare the utility's investment strategy under a permit system with its investment strategy under a fixed emissions standard. This paper examines the farm's technology investment decision both when it cannot adjust its input levels (under the non-market system) and when it can adjust its input levels (under the market system). In addition, this paper considers the effect of input supply uncertainty[1] in addition to input price uncertainty.

## III. TECHNOLOGY ADOPTION WITH A NON-MARKET ALLOCATION SYSTEM

*Stochastic Water Supply*

The aggregate supply of water to the agricultural sector fluctuates stochastically due to changes in weather and water policy. Over time, average supplies to agriculture are assumed to be falling as the growth in urban and environmental water demand exceeds the growth in agricultural demand. The aggregate supply (in units of acre-feet) is represented by the variable $W(t)$, which follows a geometric Brownian motion with negative drift

[1] Analytically, permit supply uncertainty would be analogous to water supply uncertainty.

$dW = \mu_w W dt + \sigma_w W dz_w$ .[2]

An individual farm receives an allocation of $X_0$ acre-feet at time $t$, which is a function of the aggregate supply, $W$. Suppose water is allocated according to a proportional rationing system. For $J$ water users,

$$X_0^j = \gamma^j W, \quad \text{and} \quad \sum_{j=1}^{J} \gamma^j = 1.$$

If the aggregate supply is divided into equal shares of $1/W$ acre-feet, and farm $j$ owns $K^j$ shares, then

$$\gamma^j = \frac{1}{W}(K^j).^3$$

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[2] $W = W(t)$. $z = z(t)$, etc. For notational simplicity, $t$ is not explicitly written here nor elsewhere in the paper. $W$ is known as an Ito variable. It is a function of the Wiener variable, z. A Wiener process can be thought of as a continuous time version of a random walk. The process has three key properties: First, it is a Markov process. This means that the probability distribution of future values of the process depends only on the current value, and therefore the current value is sufficient to make efficient forecasts of future values. Second, the probability distribution of the change in the process over any time interval is independent of any other non-intersecting time interval. In other words, the Wiener process has independent increments. Third, changes in the process over any finite interval are normally distributed with a mean of zero and a variance which grows linearly over the time interval. Since the variance goes to infinity over the long run, the process is nonstationary. The derivative $dz/dt = \infty$ therefore does not exist in the conventional sense. Sample paths of a Wiener process have many jagged ups and downs and are not differentiable. For more information please refer to Dixit and Pindyck, Chapter 3.

Since it does not make sense to have negative supply, the logarithm of $W$ is modeled as a Wiener process, rather than $W$ itself. Over any time interval, the change in $W$, $dW$, is lognormally distributed and the percentage change in $W$, $dW/W$, is normally distributed. $\mu_w W$ is the expected instantaneous drift rate of the Ito process, and $\sigma_w{}^2 W^2$ is the instantaneous variance rate. Thus,

$$E(dW) = \mu_w W dt$$
$$V(dW) = \sigma^2 W^2 dt.$$

Since the variance depends on $dt$, the standard deviation depends on $\sqrt{dt}$ . In the short run the volatility of the process dominates, because in the short run $\sqrt{t} >> t$ , but in the long run the trend of the process dominates, because in the long run $\sqrt{t} << t$.

[3] A proportional rationing system is used to allocate water in the Colorado-Big Thompson project. In other projects, for example the Central Valley Project in California, a priority rationing system (also known as a queuing system) is used. The function describing the relationship between $X_0$ and $W$ will be more complex with a priority rationing system, but the same principles apply.

A farm's production depends on the number of acre-feet applied per acre. Given that farm $j$ is $A^j$ acres,

$$x_0^j = \frac{1}{A^j}\left(X_0^j\right) = \frac{1}{A^j}\left(\gamma^j W\right)$$

is the number of acre-feet per acre allocated to farm $j$ at time $t$. The stochastic process of $x_0^j$ can be found using Ito's Lemma.[4] For notational simplicity, the superscript $j$ is ignored. By Ito's Lemma:

$$\begin{aligned}
dx_0 &= \frac{1}{A}\gamma dW \\
&= \frac{1}{A}\gamma\left(\mu_w W dt + \sigma_w W dz\right) \\
&= \mu_w\left(\frac{1}{A}\gamma W\right)dt + \sigma_w\left(\frac{1}{A}\gamma W\right)dz \\
&= \mu_w x_0 dt + \gamma\sigma_w x_0 dz_w \\
&= \mu_x x_0 dt + \gamma\sigma_x x_0 dz_x, \qquad \text{where } \mu_x = \mu_w,\ \sigma_x = \sigma_w \text{ and } dz_x = \mathrm{dz}_w.
\end{aligned}$$

Therefore, $x_0$ also follows a geometric Brownian motion when water is allocated according to a proportional rationing system.

*Production*

Production depends on effective water, $e_i = \alpha_i x_0$, where $\alpha_i$ indicates irrigation efficiency and $0 < \alpha_i < 1$. The farm currently uses traditional irrigation technology; however, it has the option to invest in modern irrigation technology which is more efficient

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[4] It is necessary to use Ito's Lemma because, while $W$ is continuous in time, $dz/dt$ does not exist. Given an Ito variable $x$, which follows a generalized Brownian motion with drift,

$$dx = a(x,t)dt + b(x,t)dz,$$

and a function $F(x,t)$ that is at least twice differentiable in $x$ and once in $t$, by Ito's Lemma

$$dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial x}dx + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}(dx)^2.$$

(See Dixit and Pindyck, p. 79.) In this case, since $\frac{\partial x_0^j}{\partial t} = 0$ and $\frac{\partial^2 x_0^j}{\partial W^2} = 0$, $dx_0^j = \frac{\partial x_0^j}{\partial W}dW.$

(it produces more output with a given amount of water). Adoption of the modern technology requires payment of an irreversible cost *I*. The traditional irrigation technology is represented by $i = 1$ and the modern technology is represented by $i = 2$. Thus, $\alpha_1 < \alpha_2$. The farm's instantaneous production function is quadratic, and water is the only input:

$$y_i = a + b(\alpha_i x_0) - c(\alpha_i x_0)^2 \quad i = 1,2,$$

where $a, b$ and $c$ are constants. Letting $b_i = b\alpha_i$ and $c_i = c\alpha_i^2$, the production function can be written as

$$y_i = a + b_i x_0 - c_i x_0^2 \quad i = 1,2.$$ [5]

Figure 1 illustrates two quadratic production functions, one using traditional and other using modern technology. With the quadratic production function, the marginal product of water eventually becomes negative. I assume that there is free disposal and that the farm stops applying water once the marginal product reaches zero. When $x_{0i}^{\max}$ is applied, the marginal product is zero for technology *i*. If $x_0 > x_{0i}^{\max}$, the farm throws away its excess water. One can solve for $x_{0i}^{\max}$ by differentiating the production function with respect to $x_0$ and setting the first order condition equal to zero.

$$\partial y_i / \partial x_0 = b_i - 2c_i x_{0i}^{\max} = 0$$

Output is maximized when the farm applies

$$x_{0i}^{\max} = \frac{b_i}{2c_i} = \frac{b}{2c\alpha_i} \quad i = 1,2.$$

Since $\alpha_1 < \alpha_2$, it follows that $x_{02}^{\max} < x_{01}^{\max}$, and thus production is maximized using less water with the modern technology than the traditional technology. However, the maximum output achievable is independent of the technology used; therefore, the areas under the two

[5] The concept of effective water is taken from Caswell *et al.* (1986, 1990). While a more general production function may be used in future versions of this paper, Caswell *et al* find that yield can be approximated reasonably as a quadratic function of effective water. They conduct a numerical simulation of cotton production in the San Joaquin Valley using a quadratic production function with the following parameter values

$$y_i = -1{,}589 + 2{,}311(\alpha_i x_i) - 462(\alpha_i x_i)^2.$$

production functions in Figure 1 are equal. Plugging in $x_{0i}^{\max}$ and simplifying, maximum output with either technology is

$$y^{\max} = a + {b^2}\!/_{4c} \quad i = 1,2.$$

If $a < 0$, $y_i$ will not become positive until a minimum amount of water is applied. Using the quadratic formula, one can solve for the values of $x_0$ for which $y_i$ is zero. Let $x_{0i}^L$ and $x_{0i}^H$ be the low and high values of $x_0$ that solve the equation $a + b_i x_0 - c_i x_0^2 = 0$. Then

$$x_{0i}^L = \frac{b - \sqrt{b^2 + 4ac}}{2c\alpha_i}$$

$$x_{0i}^H = \frac{b + \sqrt{b^2 + 4ac}}{2c\alpha_i}$$

The high values can be ignored since $x_{0i}^H > x_{\alpha}^{\max}$ and the farm never uses more than $x_{0i}^{\max}$. The low values are relevant. Assuming $a < 0$, and given $\alpha_1 < \alpha_2$, then $x_{02}^L < x_{01}^L$ as illustrated. In other words, less water is required for the first unit of production with modern technology than with traditional technology.

*The Profit Functions*

Since $x_0$ is exogenously determined, the farm does not have to solve a maximization problem. Assuming $x_0 > x_{\alpha}^L$ the farm's only decisions are which technology to use and whether or not to use its entire supply of water. The profit function for each technology is defined for three regions of $x_0$:

(1) $\pi_i(x_0) = 0$ if $0 < x_0 \le x_{\alpha}^L$ $i = 1,2$

(2) $\pi_i(x_0) = P\left[a + b_i x_0 - c_i x_0^2\right]$ if $x_{0i}^L < x_0 < \dfrac{b}{2c\alpha_i}$ $i = 1,2$

(3) $\pi_i(x_0) = P\left[a + {b^2}\!/_{4c}\right]$ if $x_0 \ge \dfrac{b}{2c\alpha_i}$ $i = 1,2.$

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Cotton yield is measured in pounds per acre.

In region (1) production is zero, in region (2) production is positive but less than $y^{max}$, and in region (3) production equals $y^{max}$. The farm is assumed to be a price taker in the output market. In order to concentrate on the effect of input supply uncertainty, the output price $P$ is assumed to be fixed.

*The Investment Decision*

Suppose at first that the farm does not have the option to switch to the modern technology. Then the expected present value of the farm at $t = 0$, given an initial value of $x_0$ is

$$\Pi_1(x_0) = E\left\{\int_0^{\infty} \pi_1(x_\alpha) e^{-\rho t} dt\right\},$$

where the profit flow with the traditional technology defined as above. Now suppose the farm has the option to switch to the modern technology. If the farm switches to the modern technology it must pay a one time sunk cost of $I$.[6] The increase in profit with the modern technology would be

$$\Delta\pi(x_0) = \pi_2(x_0) - \pi_1(x_0).$$

The increase in profit is the value of the investment at time $t$. $\Delta\pi(x_0)$ is defined over five regions, which depend on $x_0$:

| | |
|---|---|
| (1) $\Delta\pi(x_0) = 0$ | if $x_0 \le x_{02}^{min}$ |
| (2) $\Delta\pi(x_0) = P\left[a + b_2 x_0 - c_2 x_0^2\right]$ | if $x_{02}^{min} < x_0 \le x_{01}^{min}$ |
| (3) $\Delta\pi(x_0) = P\left[(b_2 - b_1)x_0 - (c_2 - c_1)x_0^2\right]$ | if $x_{01}^{min} < x_0 < b/2c\alpha_2$ |
| (4) $\Delta\pi(x_0) = P\left(b^2/4c\right) - P\left[b_1 x_0 - c_1 x_0^2\right]$ | if $b/2c\alpha_2 \le x_0 < b/2c\alpha_1$ |
| (5) $\Delta\pi(x_0) = 0$ | if $x_0 \ge b/2c\alpha_1$. |

In region (1) $x_0$ is so low that production is zero with either technology. In region (2) production is positive with the modern technology and zero with the traditional technology.

[6] Marginal operating costs for each technology are assumed to be zero. This restriction could be lifted in future versions of the paper.

In region (3) production is positive but less than $y^{max}$ with both technologies. In region (4) production is less than $y^{max}$ with the traditional technology but equal to $y^{max}$ with the modern technology. In region (5) production is equal to $y^{max}$ with both technologies. Figure 2 illustrates $\Delta\pi(x_0)$. It reaches its maximum in region (3) where production is positive but less than $y^{max}$ with both technologies. Define $x_0^*$ to be the value of $x_0$ that maximizes the profit-difference function. $x_0^*$ is found by setting the derivative of the function in region (3) equal to zero,

$$\partial\Delta\pi(x_0^*)\Big/\partial x_0 = Pb(\alpha_2 - \alpha_1) - 2Pc(\alpha_2 - \alpha_1)^2 x_0^* = 0.$$

Solving,

$$x_0^* = \frac{b}{2c(\alpha_2 + \alpha_1)}.$$

Then,

$$\partial\Delta\pi(x_0)\Big/\partial x_0 \begin{array}{ll} \geq 0 & \text{if } x_0 < x_0^* \\ \leq 0 & \text{if } x_0 > x_0^*. \end{array}$$

Assuming that the initial value of $x_0$ is greater than $x_0^*$, $\Delta\pi(x_0)$ will be decreasing in $x_0$. If the farm switches to the modern technology at time $t$, the payoff in terms of the present value of the increase in profit is

$$\Delta\Pi(x_0) = E\left\{\int_t^\infty \Delta\pi(x_{0t}) e^{-\rho t}\, dt\right\}.$$

For large $x_0$, $\Delta\Pi(x_0)$ will be less than the fixed cost of investment and therefore the option to switch technologies is said to be "out of the money." However, the option to switch technologies may become "in the money" when $x_0$ drops to a threshold level $\underline{x}_0$.

*The Value of the Option to Invest*

Define $F(x_0)$ to be the value of the option to switch technologies. $F(x_0)$ can be calculated using a dynamic programming approach. Since there is no payoff (in terms of increased profit) until the investment in modern technology is made, the only return to holding the option is its capital appreciation. In the region $(\underline{x}_0, \infty)$, in which the farm holds onto its opportunity to invest, the Bellman equation is

$$\rho F(x_0)dt = E[dF(x_0)].$$

Over a time interval *dt*, the total expected return on the investment opportunity, $\rho F dt$, is equal to its expected rate of capital appreciation. Using Ito's Lemma, *dF*, can be expanded as follows

$$dF(x_0) = F'(x_0)dx_0 + \tfrac{1}{2}\sigma_x^2 F''(x_0)(dx_0)^2.$$

Substituting in the stochastic process for $dx_0$ and noting that $E(dz) = 0$,

$$E[dF(x_0)] = \mu_x x_0 F'(x_0)dt + \tfrac{1}{2}\sigma_x^2 x_0^2 F''(x_0)dt.$$

Dividing through by *dt*, $F(x_0)$ satisfies the following differential equation

$$\tfrac{1}{2}\sigma_x^2 x_0^2 F''(x_0) + \mu_x x_0 F'(x_0) - \rho F(x_0) = 0,$$

subject to the boundary conditions,

$$F(0) = 0$$
$$F(\underline{x}_0) = \Delta\Pi(\underline{x}_0) - I$$
$$F'(\underline{x}_0) = \Delta\Pi'(\underline{x}_0)$$

The threshold allocation level $\underline{x}_0$ which triggers investment must be found as part of the solution. The general solution for the value of the option is

$$F(x_0) = A_1 x_0^{\beta_1} + A_2 x_0^{\beta_2},$$

where $\beta_1$ and $\beta_2$ are the positive and negative roots of the fundamental quadratic

$$\tfrac{1}{2}\sigma_x^2 \beta(\beta - 1) + (\rho - \delta)\beta - \rho = 0,$$

and the constants $A_1$ and $A_2$ remain to be determined. Since the value of the option to switch technologies goes to zero as $x_0$ goes to infinity, the coefficient on the positive root $A_1$ must be set equal to zero to prevent the term with the positive root from exploding. The general solution for the value of the option thus reduces to

$$F(x_0) = A_2 x_0^{\beta_2}.$$

*The Threshold of Investment*

Substituting in the solution for the value of the option to invest, the boundary conditions at the investment threshold become

$$A_2 \underline{x}_0^{\beta_2} = \Delta\Pi(\underline{x}_0) - I$$
$$\beta_2 A_2 \underline{x}_0^{\beta_2 - 1} = \Delta\Pi'(\underline{x}_0)$$

These are known as the value matching and smooth pasting conditions. The value matching condition states that, at the threshold, the value of the farm's option to invest equals the value of the investment minus the cost of the investment. The smooth pasting condition states that, at the threshold, the increase in the value of the option associated with a decrease in $x_0$ equals the increase in the expected present value of the investment associated with a decrease in $x_0$. Combining the value matching and smooth pasting conditions, one can solve for the function which defines the threshold $\underline{x}_0$

$$\Delta\Pi(\underline{x}_0) - \frac{\underline{x}_0}{\beta_2}\Delta\Pi'(\underline{x}_0) = I.$$

*The Value of the Farm with the Option to Invest*

$\Pi_1(x_0)$ was the expected present value of the farm at $t = 0$ assuming it did *not* have the option to invest in the modern technology. If one now considers the value of the farm's option to switch technologies, the expected present value of the farm at $t = 0$ becomes

$$\Pi(x_0) = E\left\{\int_0^\infty \pi_1(x_{0t})e^{-\rho t}dt\right\} + A_2(p)^{\beta_2}.$$

For $x_0 < \underline{x}_0$ the farm holds on to its option to invest and uses the traditional technology, and for $x_0 \geq \underline{x}_0$ the farm exercises its option and produces with the modern technology. At the threshold, $\underline{x}_0$, the value of the option to invest equals the expected increase in profits minus the investment cost, so the expected present value of the farm at the time of investment is

$$\begin{aligned}
\Pi(\underline{x}_0) &= E\int_t^\infty \pi_1\left(\underline{x}_{0t}\right)e^{-\rho t}\,dt + \Delta\Pi(\underline{x}_0) - I \\
&= E\int_t^\infty \pi_1\left(\underline{x}_{0t}\right)e^{-\rho t}\,dt + E\int_t^\infty \Delta\pi\left(\underline{x}_{0t}\right)e^{-\rho t}\,dt - I \\
&= E\int_t^\infty \pi_1\left(\underline{x}_{0t}\right)e^{-\rho t}\,dt + E\int_t^\infty \pi_2\left(\underline{x}_{0t}\right)e^{-\rho t}\,dt - E\int_t^\infty \pi_1\left(\underline{x}_{0t}\right)e^{-\rho t}\,dt - I \\
&= E\int_t^\infty \pi_2\left(\underline{x}_{0t}\right)e^{-\rho t}\,dt - I\,.
\end{aligned}$$

Thus, the expected present value of the farm at the time of investment is just the expected present value of the stream of profits associated with the modern technology, less the fixed cost of investment.

### *The Effect of Greater Uncertainty*

If the degree of uncertainty $\sigma_x$ increases, the farm will wait until its allocation drops to a lower level before it invests. An increase in uncertainty lowers the investment threshold $\underline{x}_0$ in two ways. First, an increase in $\sigma_x$ increases $\beta_2$, which causes the option value term in the threshold equation to increase, and this decreases the threshold $\underline{x}_0$. The second effect results from the concavity of $\Delta\Pi(x_0)$ in $x_0$. (As shown in Figure 2, as $x_0$ decreases, $\Delta\pi(x_0)$ increases at a decreasing rate until eventually, if $x_0 < x_0^*$, $\Delta\pi(x_0)$ begins decreasing.) Since $x_0$ follows a geometric Brownian motion, the variance of the distribution of $x_0$ increases as one looks farther into the future. By Jensen's Inequality, the expected value of a concave function decreases as the variance increases. Therefore, an increase in $\sigma_x$ decreases the expected present value of the profit-difference stream $\Delta\Pi(x_0)$. This discourages investment and reduces $\underline{x}_0$.

*The Effect of Increased Investment Cost*

Since $\Delta\pi(x_0)$ is only increasing for $x_0 > x_0^*$, investment will only occur if there exists a threshold allocation level $\underline{x}_0$ greater than or equal to $x_0^*$. There will be a cutoff investment cost $\overline{I}$ at which $\underline{x}_0 = x_0^*$, and for $\overline{I} > I$ investment will not occur.

## IV. TECHNOLOGY ADOPTION UNDER A MARKET SYSTEM

*Stochastic Market Price*

Now suppose the farm can trade water in a competitive market, and there are no transaction costs associated with trading. Assume the market price of water $p$ follows a geometric Brownian motion with positive drift

$$dp = \mu_p p dt + \sigma_p p dz_p,$$

and $p$ is negatively correlated with $W$, i.e. $E\left[dz_w dz_p\right] = \rho dt$ and $\rho < 0$.

*Production with Variable Input*

The farm's production function is the same as before, except now the farm can buy or sell water in order to increase or decrease its input supply in response to changes in the market price of water. Since the market is competitive, an individual farm's trading behavior is assumed to have no impact on the market price. Let $x_i$ be the amount of water the farm trades if it is using technology $i$. $x_i$ will be positive if the farm buys water or negative if the farm sells water. The farm's production function is

$$y_i = a + b\alpha_i\left(x_0 + x_i\right) - c\left[\alpha_i\left(x_0 + x_i\right)\right]^2 \quad i = 1,2.$$

With each technology, the farm chooses $x_i$ to maximize profits subject to the constraint that $x_i$ is greater than or equal to $-x_0$. In other words, the farm cannot sell more than its initial allocation. The optimization is

$$\pi_i^m(p) = \max_{x_i} P\left[a + b_i\left(x_0 + x_i\right) - c_i\left(x_0 + x_i\right)^2\right] - px_i \quad \text{s.t. } x_i \geq -x_0 \;\; i = 1,2.$$

The superscript, $m$, distinguishes this unconstrained "market" profit function from the constrained non-market profit function from section III. From the first order condition, one can solve for the optimal level of water use with technology $i$





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$$x_0 + x_i = \begin{cases} \dfrac{b}{2c\alpha_i} - \left(\dfrac{1}{2Pc\alpha_i^{\,2}}\right)p & \text{if } p < p^* \\ 0 & \text{if } p \geq p_i^* \end{cases}.$$

$p_i^*$ is the price at which the production revenue with technology *i* equals zero. Thus, when $p \geq p_i^*$ the farm will sell its entire supply of water. The amount the farm trades in the market with technology *i* is

$$x_i = \begin{cases} \dfrac{b}{2c\alpha_i} - \left(\dfrac{1}{2Pc\alpha_i^{\,2}}\right)p - x_0 & \text{if } p < p^* \\ -x_0 & \text{if } p \geq p^* \end{cases}.$$

Figure 3 illustrates the inverse demand function associated with technology *i*

$$p = Pb\alpha_i - 2Pc\alpha_i^{\,2}(x_0 + x_i).$$

The curve is kinked at $p_i^*$, the point at which the farm stops producing. When $x_0 + x_i = b/_{2c\alpha_i}$, the farm produces $y^{max}$ and, therefore, $p = 0$. Figure 4 shifts the axis by $x_0$ in order to show excess demand, *i*.e. the amount the farm trades in the market. The farm will buy (sell) with technology *i* if $p$ is less than (greater than) $p_i^0$, where

$$p_i^0 = Pb\alpha_i - 2Pc\alpha_i^{\,2}x_0.$$

Note that a change in $x_0$ does not shift the curve in Figure 3, because the farm adjusts $x_i$ to stay on the curve. The adjustment in $x_i$ can be seen in Figure 4. If $x_0$ increases, the curve in Figure 4 shifts to the left, and if $x_0$ decreases, the curve shifts to the right.

Plugging in the optimal $x_i$, profit can be calculated as a function of $p$, $x_0$ and the production parameters. The form of the profit function depends on whether the farm produces or sells all of its water

$$\pi_i^m(p, x_0) = Pa + \frac{(Pb_i - p)^2}{4Pc_i} + px_0 \qquad \text{if } p < p_i^* \quad i = 1,2$$





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$$\pi_i^m(p, x_0) = px_0 \qquad \text{if } p \geq p_i^* \qquad i = 1,2 \quad .$$

Now one can solve for $p_i^*$, the price at which the production revenue equals zero. Setting

$$Pa + \frac{(Pb_i - p)^2}{4Pc_i} = 0,$$

one finds

$$\begin{aligned} p_i^* &= P\left[b_i + 2\sqrt{-ac_i}\right] \\ &= Pb\alpha_i\left[1 + \sqrt{-ac}\big/b\right]. \end{aligned}$$

Since $a$ is assumed to be negative, the number under the square root is positive. Note that $p_1^* < p_2^*$ and thus the farm stops production at a lower price with the traditional technology than with the modern technology.

*The Investment Decision*

As in the non-market case, first suppose that the farm does not have an option to switch technologies. Then the value of the farm at $t = 0$, given initial values $p$ and $x_0$, is

$$\Pi_1^m(p, x_0) = E\left\{\int_0^\infty \pi_1^m(p_t, x_{0t})e^{-\rho t}dt\right\},$$

where $\pi_1^m(p_t, x_{0t})$ is the instantaneous profit flow with the traditional technology as defined above. Now suppose the farm has the option to switch to the modern technology. If the farm switches to the modern technology it must pay a one time sunk cost of $I$. Its increase in profit with the modern technology is

$$\Delta\pi^m(p) = \pi_2^m(p, x_0) - \pi_1^m(p, x_0).$$

$\Delta\pi^m(p)$ is the value of the investment at time $t$. It is defined over three regions, which depend on $p$.[7]

[7] In contrast, in the no-market case there were five regions, which depended on $x_0$. There were five regions in the no-market case, because if $x_0$ was large enough to allow the farm to produce $y^{max}$ with both technologies, the profit-difference equaled zero. Even though the farm could produce $y^{max}$ with less water

(1) $\Delta\pi^m(p)=\dfrac{(Pb_2-p)^2}{4Pc_2}-\dfrac{(Pb_1-p)^2}{4Pc_1}$ if $p < p_i^*$ $i=1,2$

(2) $\Delta\pi^m(p)= Pa+\dfrac{(Pb_2-p)^2}{4Pc_2}$ if $p_1^* \le p < p_2^*$

(3) $\Delta\pi^m(p)=0$ if $p \ge p_2^*$

Figure 5 illustrates $\Delta\pi^m(p)$. In region (1) the farm produces with both technologies, in region (2) the farm only produces with the modern technology and in region (3) the farm does not produce with either technology. Note that while the profit associated with a given technology depends on $x_0$, the *change in profit* is independent of $x_0$.

Setting the derivative of $\Delta\pi^m(p)$ in region (1) equal to zero and solving for $p$, one finds

$$p^{\max}=\frac{Pb\alpha_1\alpha_2}{\alpha_1+\alpha_2}=Pb\alpha_2\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right)$$

and

$$\partial\Delta\pi^m(p)\Big/\partial p \;\begin{cases} >0 & \text{if } p < p^{\max} \\ <0 & \text{if } p > p^{\max}\end{cases}$$

In region (1) $\Delta\pi^m(p)$ is increasing up until $p^{\max}$ and then decreasing thereafter. $\Delta\pi^m(p)$ is decreasing throughout region (2). Assuming that the initial value of $p$ is less than $p^{\max}$, as $p$ rises $\Delta\pi^m(p)$ will increase. The farm can switch the modern technology at any time $t$. If it switches at $t$, the payoff in terms of the present value of the increase in profit is

$$\Delta\Pi^m(p)=E\left\{\int_t^{\infty}\Delta\pi^m(p_t)e^{-\rho t}dt\right\}.$$

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with the modern technology, the extra water was worthless. Now, however, even if p is low enough to allow the farm to produce $y^{\max}$ with either technology, there is still an advantage to the modern technology because the farm can produce $y^{\max}$ with less water with the modern technology than with the traditional technology. If the farm is a buyer, its input costs will be lower and if the farm is a seller, its revenue will be higher with the modern technology. Only when the price falls to zero, does the profit difference fall to zero. Therefore, the profit function is defined over only three regions instead of five.

Over some price ranges, $\Delta\Pi^m(p)$ is less than the fixed cost of investment; therefore, the option to switch technologies is "out of the money." However, the option to switch could become "in the money" for higher values of $p$.

*The Value of the Option to Invest*

The value of the option to switch technologies can be calculated as in the non-market case. Using the same logic as before, it can be shown that the value of the option satisfies the following differential equation

$$\tfrac{1}{2}\sigma_p{}^2 p^2 F^{m''}(p) + \mu_p p F^{m'}(p) - \rho F^m(p) = 0,$$

subject to the boundary conditions

$$F^m(0) = 0$$
$$F^m(\overline{p}) = \Delta\Pi^m(\overline{p}) - I$$
$$F^{m'}(\overline{p}) = \Delta\Pi^{m'}(\overline{p}).$$

The threshold price $\overline{p}$ must be found as part of the solution. Since the *difference* in profit between the modern and traditional technologies is independent of $x_0$, this is an ordinary differential equation in $p$. The general solution for the value of the option is

$$F^m(p) = B_1 p^{\beta_1} + B_2 p^{\beta_2},$$

where $\beta_1$ and $\beta_2$ are the positive and negative roots of the fundamental quadratic. Since the value of the option to switch technologies goes to zero as $p$ approaches zero, the coefficient $B_2$ must be set equal to zero to prevent the term with the negative root from exploding. The solution thus reduces to

$$F^m(p) = B_1 p^{\beta_1}.$$

*The Investment Threshold*

Substituting in the solution for the value of the option to invest, the value matching and smooth pasting conditions are

$$B_1\overline{p}^{\beta_1} = \Delta\Pi^m(\overline{p}) - I$$
$$\beta_1 B_1\overline{p}^{\beta_1 - 1} = \Delta\Pi^{m'}(\overline{p}).$$

As before, the value matching condition states that the value of the farm's option to invest equals the value of the investment minus the cost of the investment at the threshold. The smooth pasting condition states that the increase in the value of the option associated with an increase in $p$ equals the increase in the expected present value of the investment at the threshold. Combining the value matching and smooth pasting conditions, one can solve for the function which implicitly defines the threshold $\overline{p}$

$$\Delta\Pi^m(\overline{p}) - \frac{\overline{p}}{\beta_1}\Delta\Pi^{m'}(\overline{p}) = I .$$

*The Value of the Farm with the Option to Invest*

$\Pi_1^m(p, x_0)$ was the expected present value of the farm assuming it did not have the option to invest in the modern technology. If one now considers the value of the farm's option to switch technologies, the expected present value of the farm's profits at $t = 0$ becomes

$$\Pi^m(p, x_0) = E\left\{\int_0^\infty \pi_1^m(p_t, x_{0t})e^{-\rho t}dt\right\} + B_1(p)^{\beta_1} .$$

For $p < \overline{p}$ the farm holds on to its option to invest and uses the traditional technology, and for $p \geq \overline{p}$ the farm exercises its option and produces with the modern technology. At the threshold, $\overline{p}$, the value of the option to invest equals the expected increase in profits minus the investment cost,

$$\begin{aligned}\Pi^m(\overline{p}, x_0) &= E\int_t^\infty \pi_1^m(\overline{p}_t, x_{0t})e^{-\rho t}dt + \Delta\Pi^m(\overline{p}) - I \\ &= E\int_t^\infty \pi_1^m(\overline{p}_t, x_0)e^{-\rho t}dt + E\int_t^\infty \Delta\pi^m(\overline{p}_t)e^{-\rho t}dt - I \\ &= E\int_t^\infty \pi_1^m(\overline{p}_t, x_0)e^{-\rho t}dt + E\int_t^\infty \pi_2^m(\overline{p}_t, x_0)e^{-\rho t}dt - E\int_t^\infty \pi_1^m(\overline{p}_t, x_0)e^{-\rho t}dt - I\end{aligned}$$

$$= E\int_{t}^{\infty} \pi_2^m(\overline{p}_t, x_0)e^{-\rho t}\,dt - I\,.$$

Thus, the expected present value of the farm at the time of investment is the expected present value of the stream of profits associated with production using the modern technology minus the fixed cost of investment. Notice that the allocation $x_0$ affects the expected present value of the farm, but it does not affect the timing of investment. The timing of investment only depends the time it takes for $p$ to reach $\overline{p}$.

*The Effect of an Increase in Uncertainty*

As in the non-market case, there are two effects of an increase in $\sigma_p$. First, an increase in $\sigma_p$ decreases $\beta_1$. As $\beta_1$ falls, the option value term in the threshold equation increases, and the threshold $\overline{p}$ that is required to trigger investment increases. The second effect results from the concavity of $\Delta\Pi^m(p)$ in $p$. (Note, the profit functions $\pi_i^m(p)$ are *convex* in $p$: as $p$ increases, $\pi_i^m(p)$ decrease at a decreasing rate. However, the *difference* in profit, $\Delta\pi^m(p)$, is *concave* in $p$: as $p$ increases, $\Delta\pi^m(p)$ increases at a decreasing rate until eventually, if $p > p^{\max}$, $\Delta\pi^m(p)$ begins decreasing.) By the properties of the geometric Brownian motion, the variance of the distribution of $p$ increases as one looks farther into the future. By Jensen's Inequality, the expected value of a concave function decreases as the variance increases. Therefore an increase in $\sigma_p$ decreases $\Delta\Pi^m(p)$. This discourages investment and causes $\overline{p}$ to increase.

*The Effect of Increased Investment Cost*

If $\Delta\pi^m(p)$ were everywhere increasing in $p$, eventually $p$ would rise high enough to trigger investment, *i.e.* for each $I$ one could solve for $\overline{p}$. However, since $\Delta\pi^m(p)$ is only increasing for $p < p^{\max}$, investment will only occur if there exists a threshold price $\overline{p}$ less than $p^{\max}$. It follows that there will be a cutoff investment cost $\overline{I}$, at which $\overline{p} = p^{\max}$, and for $I > \overline{I}$ investment will never occur.

## V. INVESTMENT WITH AND WITHOUT A WATER MARKET

*Static Example of Technology Adoption*

It is often claimed that farms will have greater incentive to adopt modern irrigation technology if they have access to a water market. In fact, while a farm's incentive to adopt modern technology may increase, it is also possible that its incentive may decrease. Farms which might have adopted modern technology under a non-market water allocation system, may be able to delay adoption when they have the option to buy water in a market. It is more accurate to say that access to water markets will result in more *efficient* technology adoption, not necessarily more technology adoption.

The comparison between non-market and market water allocation systems is closely related to the comparison between a fixed pollution standard and a tradable emission permit system. As discussed in section II, Malueg (1989) uses a static graphical analysis to demonstrate that a shift from a fixed standard to a tradable permit system may increase or decrease the incentive for firms to switch from a high-cost to a low-cost pollution abatement technology. While Malueg's framework is static with no uncertainty, it is illustrative to work through a similar logic for the water model before analyzing the technology adoption decision within the stochastic dynamic framework employed in this paper.

Reducing the production functions in sections III and IV to a single time period, I compare the increase in profit associated with adopting modern irrigation technology under the non-market and market systems. I show that, for a given initial allocation $x_0$, there exists a price, $p_s$, for which the gains from modern technology adoption are the same under both systems. For $p$ greater than (less than) $p_s$, I show that the gains from the modern technology are greater (less) under a market system than a non-market system. To demonstrate this, I first compare the value of switching to a market system when only the traditional technology is available, and the value of switching to a market system when only the modern technology is available. I then use these results to compare the value of switching technologies under a market versus non-market system. Thus initially I hold the technology type fixed and allow the *allocation system* to change, and then I hold the allocation system fixed and allow the *technology* to change.

*Non-Market vs. Market Allocation*

Suppose only the traditional technology is available, and one wants to know the value of switching to a market system. The increase in profit under a market system depends on the amount by which the farm's initial allocation $x_0$ deviates from the amount it would optimally purchase if it had access to a market. For each $x_0$, there exists a price, $p_1^0$, at which $x_0$ is the optimal input quantity. Referring back to Figure 4, $p_1^0$ is the price at which the excess demand for water equals zero given an initial allocation of $x_0$. For $p > p_1^0$, the farm will sell some of its initial allocation, and for $p < p_1^0$, the farm will by more water. Let $\Delta\pi_1 = \pi_1^m(p, x_0) - \pi_1(x_0)$ be the change in profit with the traditional irrigation, where $m$ distinguishes the unconstrained market profit function from the constrained non-market profit function as before, and $i = 1$ is the traditional technology. The change in profit is just the profit with the traditional technology under the market system less the profit with the traditional technology under the non-market system. The relationship between $\Delta\pi_1$ and $p$ is illustrated by the solid line in Figure 6. If the price equals $p_1^0$, the farm uses $x_0$ under both the non-market and the market systems, so $\Delta\pi_1 = 0$. If $p$ is greater than or less than $p_1^0$, $\Delta\pi_1 > 0$ because with the market system the farm can adjust its input level in response to changes in the price. For $p$ greater than $p_1^0$, the farm sells water, and for $p$ less than $p_1^0$, the farm buys water. This is just a demonstration of LeChatelier's principle.

If the farm were using the modern technology ($i = 2$) instead of the traditional technology, there would exist another price, $p_2^0$, at which $x_0$ is the optimal input quantity. And again, for each $x_0$, there is a different $p_2^0$. With the modern technology, the farm's derived demand for water would pivot up and to the right as shown in Figure 7. The intercept of the demand curves occurs at $(x_i^*, p^{\max})$. For prices less than $p^{\max}$, the farm demands less with the modern technology than with the traditional technology at every price. Alternatively, the farm is willing to pay more for a given amount with the traditional technology.[8] Therefore, if $x_0^*$ is less than zero (i.e. the intercept is to the left of the vertical

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[8] By setting the demand function with traditional technology equal to the demand function with modern technology, one can solve for the price, $\hat{p}$, at which $x_0 + x_1$ equals $x_0 + x_2$:

$$\frac{b}{2c\alpha_1} - \left(\frac{1}{2Pc\alpha_1{}^2}\right)\hat{p} = \frac{b}{2c\alpha_2} - \left(\frac{1}{2Pc\alpha_2{}^2}\right)\hat{p}.$$

Solving,

access), $p_1^0$ is greater than $p_2^0$ because the farm is willing to pay more for $x_0$ with the traditional technology than with the modern technology.[9] When the farm uses the modern technology, the increase in profit associated with the market system is $\Delta\pi_2 = \pi_2^m(p, x_0) - \pi_2(x_0)$. The dotted line in Figure 6 illustrates the relationship between $\Delta\pi_2$ and $p$. At $p_2^0$, the farm uses $x_0$ under both the non-market and market systems so $\Delta\pi_2 = 0$, and as p moves above or below $p_2^0$, $\Delta\pi_2 > 0$.

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$$\hat{p} = \frac{Pb\alpha_1\alpha_2}{\alpha_1 + \alpha_2} = p^{\max},$$

so the price at which $x_0 + x_1$ equals $x_0 + x_2$ is the same price at which $\Delta\pi^m(p)$ is maximized. If $p < p^{\max}$, then the farm uses more with the traditional technology and if $p > p^{\max}$, the farm uses more with the modern technology. If adoption occurs, it occurs when $\overline{p} < p^{\max}$, in the range where $\Delta\pi^m(p)$ is still increasing. Therefore, at the point of adoption, the farm must use more with the traditional technology than with the modern technology $(x_1 > x_2)$.

At $p = p^{\max}$, the farm will use

$$x_0 + x_i = \frac{b}{2(\alpha_1 + \alpha_2)}$$

with each technology. When $p < p^{\max}$, the farm will use more than this with each technology but use relatively more with the traditional technology than with the modern technology. Given that $\overline{p} < p^{\max}$ is the threshold price, at the switch point, the farm will use

$$x_0 + x_i = \frac{b}{2c\alpha_i} - \left(\frac{1}{2Pc\alpha_i^2}\right)\overline{p} \qquad i = 1,2.$$

The farm buys with technology $i$ at the switch point if

$$\overline{p} < Pb\alpha_i - 2Pc\alpha_i^2 x_0$$

and sells if

$$\overline{p} > Pb\alpha_i - 2Pc\alpha_i^2 x_0.$$

[9] If $x_i^*$ is greater than zero (*i.e.*, the intercept of the demand curves occurs to the right of the vertical axis), $p_1^0 < p_2^0$. However, this example assumes that $x_i^*$ is less than zero because it is more intuitive to discuss the range for which the modern technology decreases water use, i.e., for a given price, the farm demands less

Ignoring the fixed costs of adopting the modern technology for now, one can compare the increase in profit associated with switching to the market system with the traditional versus the modern technologies. For the initial allocation $x_0$, $\Delta\pi_1 = \Delta\pi_2$ when the price equals $p_s$. Each new allocation level $x_0$ will have a different $p_s$. At $p_s$, the farm buys water with the traditional technology and sells water with the modern technology. For $p$ greater than (less than) $p_s$, $\Delta\pi_2$ is greater than (less than) $\Delta\pi_1$. Substituting in the definitions for $\Delta\pi_1$ and $\Delta\pi_2$,

$$\pi_2^m(p,x_0) - \pi_2(x_0) > \pi_1^m(p,x_0) - \pi_1(x_0) \qquad \text{for } p > p_s$$
$$\pi_2^m(p,x_0) - \pi_2(x_0) \leq \pi_1^m(p,x_0) - \pi_1(x_0) \qquad \text{for } p \leq p_s.$$

*Traditional vs. Modern Technology*

The above inequalities compare the effect of changing the water *allocation regime* with a given technology. They can be rearranged to obtain the following inequalities which compare the effect of changing the *technology* within a given water allocation regime.

$$\pi_2^m(p,x_0) - \pi_1^m(p,x_0) > \pi_2(x_0) - \pi_1(x_0) \qquad \text{for } p > p_s$$
$$\pi_2^m(p,x_0) - \pi_1^m(p,x_0) \leq \pi_2(x_0) - \pi_1(x_0) \qquad \text{for } p \leq p_s$$

The increase in profit associated with the modern technology is greater (less) under a market system than a non-market system if $p$ is greater (less) than $p_s$. In other words, the market increases (decreases) the farm's incentive to adopt modern technology depending on whether price is greater (less) than $p_s$. At $p_s$, the farm buys with the traditional technology and sells with the modern technology. At a sufficiently high price, the farm will sell with both the traditional and the modern technology, and likewise, at a sufficiently low price, the farm will buy with both the traditional and modern technology. If the farm is a seller, the market increases the farm's incentive to adopt modern technology, and if the farm is a buyer, the market decreases the farm's incentive to adopt modern technology. This result is analogous to Maleug's.[10]

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with the modern technology. If $x_i^*$ were greater than zero, the logic of this section would still hold but the results would be reversed.

[10] As discussed in the previous footnote, these results assume that the intercept of the excess demand curves occurs to the left of the vertical axis and therefore $p_1^0 < p_2^0$. They also assume that technology adoption occurs in the region to the right of the intercept of the two demand curves, i.e. for $\overline{p} < p^{\max}$. This intuition was discussed in section IV.

Whether the farm actually will adopt the modern technology under the non-market or market system depends on the fixed cost of adoption. Using the notation from sections III and IV to simplify the expressions,

$$\Delta\pi(x_0) = \pi_2(x_0) - \pi_1(x_0) \quad \text{and}$$

$$\Delta\pi^m(p) = \pi_2^m(p, x_0) - \pi_1^m(p, x_0).$$

Notice, once again, that while the profit with either technology under the market system is a function of $x_0$, the difference in profit $\Delta\pi^m(p)$ is only a function of $p$. The farm will adopt the modern technology under the non-market system if $\Delta\pi(x_0) \geq I$ where $I$ is the fixed cost of adoption. Likewise, the farm will adopt the modern technology under the market system if $\Delta\pi^m(p) \geq I$. In the static example, there is no option value of waiting, so the adoption rule is the traditional present-value rule.

*Dynamic Framework*

In the static example above, the effect of the market on technology adoption was determined by comparing the change in profit under the market system, $\Delta\pi^m(p)$, with the change in profit under the non-market system $\Delta\pi(x_0)$. This is a relatively easy comparison, given that $p$ and $x_0$ are fixed in a deterministic static example. In contrast, in the dynamic model used elsewhere in this paper, $x_0$ and $p$ are assumed to evolve stochastically over time. An analysis of the effect of the market on technology adoption must examine the expected present value of modern technology adoption, due to the increase profit in all future time periods, under the non-market and market systems. As defined previously, $\Delta\Pi(x_0)$ is the expected present value of switching technologies at time $t$ under the non-market system given that the allocation level at time $t$ is $x_0$, and $\Delta\Pi^m(p)$ is the expected present value of switching technologies at time $t$ under the market system given that the price at time $t$ is $p$. One can ask two related but different questions: First, at a given $t$, is $\Delta\Pi(x_0)$ or $\Delta\Pi^m(p)$ larger? Second, will a given farm invest in modern technology earlier with a non-market or market system?

The first question is most directly related to the static analysis above. As in the static model, it can be shown that there is a price $p_s$ at which $\Delta\Pi(x_0) = \Delta\Pi^m(p)$.

Although $\Delta\Pi^{m}(p)$ is independent of $x_0$, $p_s$ is a function of $x_0$ because $\Delta\Pi(x_0)$ is a function of $x_0$.[11] Assuming that $p_s$ lies in the range of prices, $0 \leq p < p^{max}$, for which $\Delta\Pi^{m}(p)$ is increasing,

$$\left.\frac{\partial\Delta\Pi^{m}(p)}{\partial p}\right|_{p_s} > 0.$$

Therefore,

$$\begin{aligned} \Delta\Pi(x_0) &< \Delta\Pi^{m}(p) && \text{for } p < p_s \\ &\geq && \text{for } p \geq p_s. \end{aligned}$$

This means that given an allocation at time *t* of $x_0$, the expected present value of switching technologies at time *t* is greater (less) with a market system than with a non-market system if the price at time *t* is greater (less) than $p_s$. The above comparison, however, does not address the question of whether it is *optimal* to invest under either system. The second question pertains to the timing of investment.

*The Timing of Investment*

Given a non-market allocation system, it was shown that the farm will invest in modern technology when $x_0$ falls to a critical level $\underline{x}_0$. By the value matching condition, when the allocation equals $\underline{x}_0$, the value of the option to invest $F(\underline{x}_0)$ equals the expected present value of switching technologies $\Delta\Pi(\underline{x}_0)$ less the cost of investment *I*. Given a market allocation system, it was shown that the farm will invest in modern technology when *p* rises to a critical level $\overline{p}$. At $\overline{p}$, the option to invest $F^{m}(\overline{p})$ is equal to the expected present value of switching technologies $\Delta\Pi^{m}(\overline{p})$ minus the cost of investment *I*. Whether a farm will invest earlier under a non-market or a market system depends on the time paths of $x_0$ and *p*. More specifically, it depends on which threshold, $\underline{x}_0$ or $\overline{p}$, is reached first.

As an exercise, suppose a farm is cloned so there are two identical farms, named Farm *A* and Farm *F*. Suppose each farm receives the same allocation of water $x_0$, but

[11] Likewise, $\Delta\Pi^{m}$ is independent of $x_0$, but $p_s$ is a function of $x_0$ since $\Delta\Pi^{NM}$ is a function of $x_0$.

Farm $F$ has access to a distribution system which allows it to trade freely in a market while Farm $A$ does not. Farm $A$ is the "Autarkic farm" and Farm $F$ is the "Free-trade farm." At any time $t$, one can calculate $(x_0, p)$, the allocation to each farm and the market price. Figure 8 shows the thresholds that trigger action in $(x_0, p)$ space. Farm $A$ will invest when its allocation falls to $x_0 = \underline{x}_0$, and Farm $F$ will invest when the price of water rises to $p = \overline{p}$. If the point $(x_0, p)$ begins in quadrant 4, both farms use the traditional technology. Cases I - III examine different time paths for $(x_0, p)$. In case I, $x_0$ falls while $p$ stays relatively constant causing the point $(x_0, p)$ to hit the threshold $x_0 = \underline{x}_0$ and move into quadrant 3. At the threshold,

$$\Delta\Pi^m(\overline{p}) - B_1(\overline{p})^{\beta_1} < \Delta\Pi(\underline{x}_0) - A_2(\underline{x}_0)^{\beta_2} = I,$$

so Farm $A$ invests while Farm $F$ holds onto its option to invest. In case II, $x_0$ remains relatively constant while $p$ falls. The point $(x_0, p)$ hits the threshold $p = \overline{p}$ and moves into quadrant 2, causing Farm $F$ to invest while Farm $A$ holds onto its option to invest. At the threshold,

$$\Delta\Pi(\underline{x}_0) - A_2(\underline{x}_0)^{\beta_2} < \Delta\Pi^m(\overline{p}) - B_1(\overline{p})^{\beta_1} = I.$$

Farm $A$ and Farm $F$ will not adopt at the same time unless $x_0$ falls and $p$ rises such that $(x_0, p)$ hits the intersection of the threshold curves and moves directly into quadrant 1 as shown in case III. When the thresholds are hit simultaneously,

$$\Delta\Pi(\underline{x}_0) - A_2(\underline{x}_0)^{\beta_2} = \Delta\Pi^m(\overline{p}) - B_1(\overline{p})^{\beta_1} = I.$$

*The Correlation Between Individual Supply and Price*

In section IV, the market price of water $p$ was assumed to be negatively correlated with the aggregate supply of water $W$, i.e. $E[dz_w dz_p] = \rho dt$ and $\rho < 0$. In section III the relationship between the aggregate water supply and individual water supply was assumed to be proportional. Given the assumption of proportional allocation, $\sigma_x = \sigma_w$, and therefore the correlation between *individual* supply and the price is also $\rho$. If instead of a proportional allocation system, there is a priority allocation system (also known as a queuing system), this relationship will not hold for all individuals. Under a priority system, the supply of senior rights holders may vary less than the aggregate supply $(\sigma_x^s < \sigma_w)$ while the supply of junior rights holders may vary more than the aggregate supply $(\sigma_x^j > \sigma_w)$. In this case, the correlation between individual supply and the price will depend on the seniority of a farm's water rights.

Continuing with the example above, the actual path of $(x_0, p)$ will depend on whether Farms $A$ and $F$ hold junior or senior water rights. If they hold junior water rights, the percentage reduction in $x_0$ may be larger than the percentage reduction in the aggregate water supply, where the aggregate supply,

$$W = x_0^A + x_0^F + \sum_{j=1}^{J} x_0^j ,$$

is the sum of supplies to all J + 2 water users. For example, $x_0^A$ and $x_0^F$ may each be reduced by 50% in a drought year even if the aggregate supply only falls by 25%. Since the market price is a function of the aggregate supply, the increase in $p$ might not be as great as the reduction in $x_0$, in which case Farm $A$ may adopt the modern technology before Farm $F$ (case I). If instead Farms $A$ and $F$ hold senior water rights, $x_0^A$ and $x_0^F$ may remain relatively constant even if the aggregate supply falls. In this case, $p$ might rise while $x_0$ remains relatively constant, and Farm $F$ may adopt before farm $A$ (case II).

## VI. TRANSACTION COSTS

This paper has considered two polar cases: a non-market system and a market system. The non-market system implicitly assumes that the transaction costs of trading are infinite. The market system assumes that the transaction costs of trading are zero. In reality, most market transactions involve some transaction costs and this is especially true with water markets. Even with institutional reforms, the fixed costs of market administration and enforcement may be high. In addition, if the market is structured to allow decentralized, bilateral trades, search and negotiation costs may be high. Water transactions may also generate negative externalities due to reductions in return flows and income in the basin of origin. If traders are forced to internalize these externalities through a tax per unit of water traded, the tax will be viewed by the traders as a marginal transaction cost. With either fixed or marginal transaction costs, there will be a range of water market prices for which the farm does not participate in the market. In the case of fixed costs, even if the marginal value product of water is greater (less) than the price, the farm will not buy (sell) water if the total gains from trade are less than the transaction costs. With marginal transaction costs, there will be a wedge between the price received by the buyer and the seller. Stavins explores the case of marginal transaction costs in a tradable permit market.[9]

If there are marginal transaction costs, there will be multiple technology investment thresholds depending on whether the farm buys, sells or does not participate in the market. For example, if the farm buys with the traditional and sells with the modern it will have one investment threshold, and if it buys with the traditional and does not participate with the modern it will have another. Fixed transaction costs could be modeled as a one-time market access fee. If the farm pays a fixed cost $F$ it can gain market access and then trade freely. With a fixed entry cost, there will be different possible paths: the farm may adopt modern technology and then invest in market access, it may invest in market access and then adopt modern technology, it may do both simultaneously, it may never adopt or invest in market access, and so on. Future versions of this paper will explore the effect of fixed and marginal transaction costs on technology adoption in more detail.

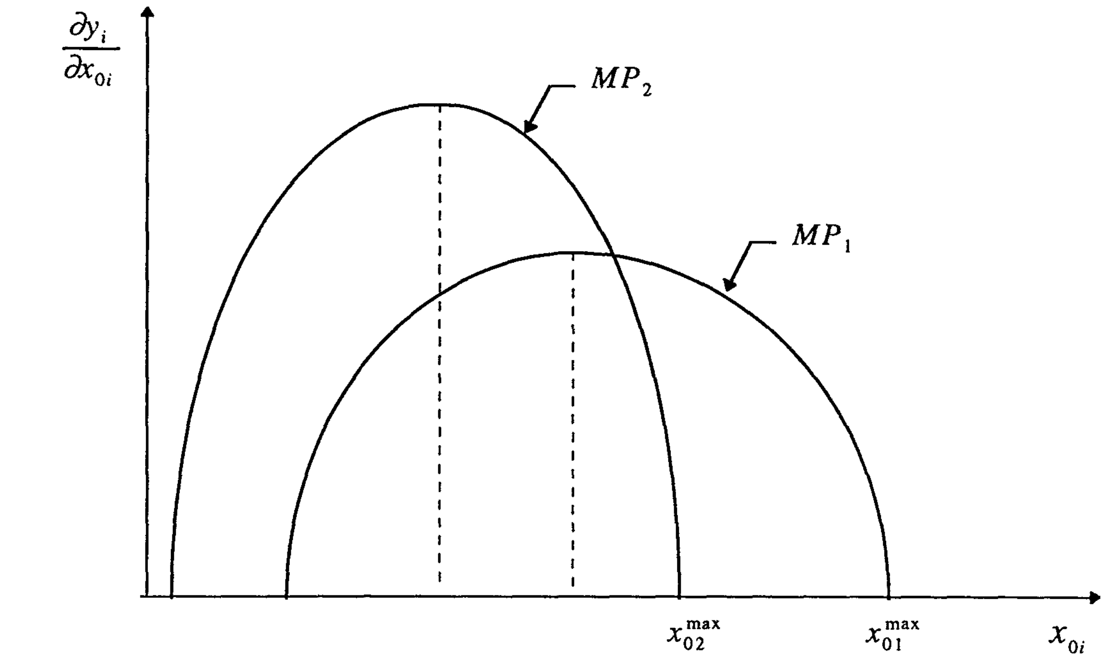


**FIGURE 1.** *Marginal productivity of water with traditional and modern technology. Area under each curve is $y^{max}$.*

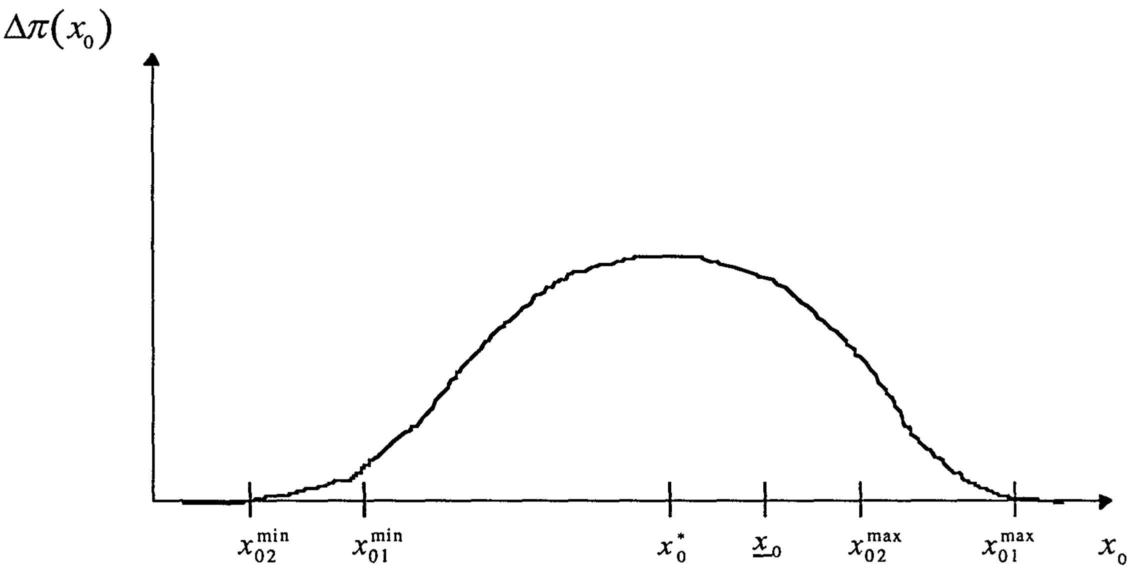


**FIGURE 2.** *The difference in profit between modern and traditional technology with a non-market system. The investment threshold is $\underline{x}_0$.*





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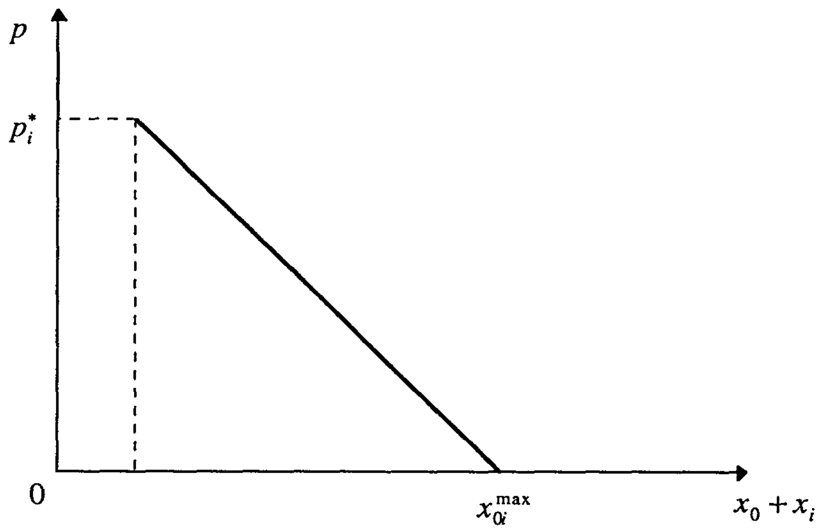


**FIGURE 3.** *Inverse demand with technology i.*

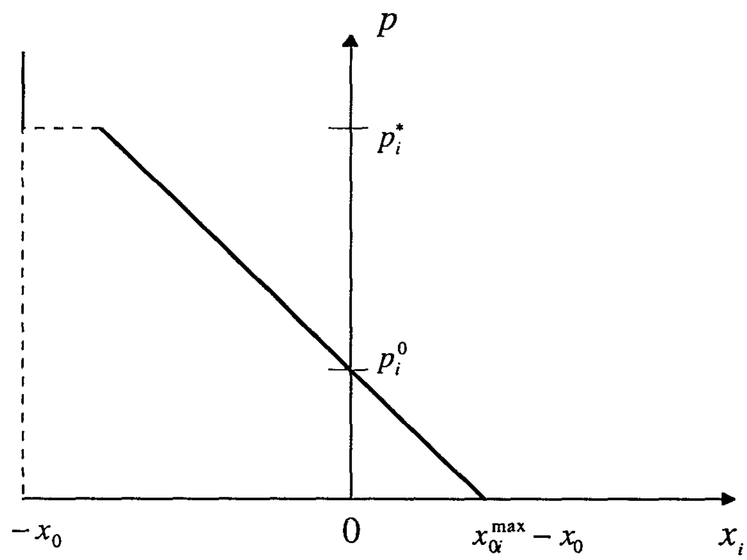


**FIGURE 4.** *Excess demand with technology i.*

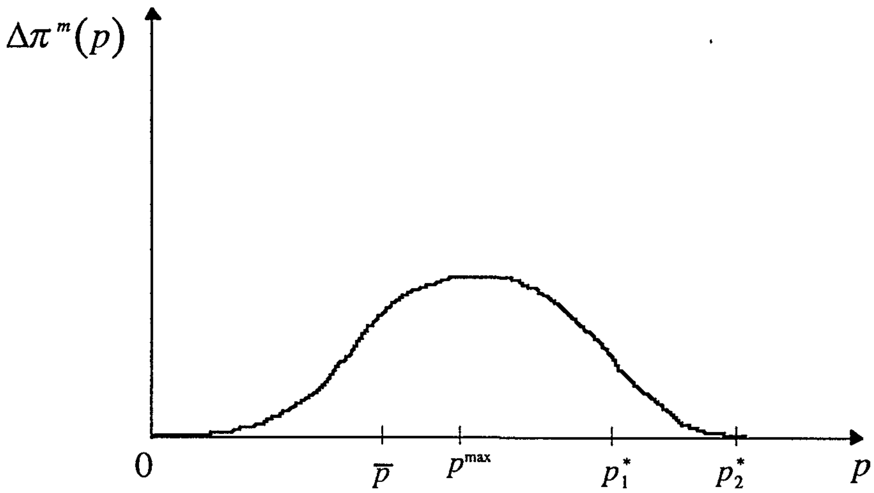


**FIGURE 5.** *The difference in profit between modern and traditional technology with a market system. The investment threshold is* $\overline{p}$.

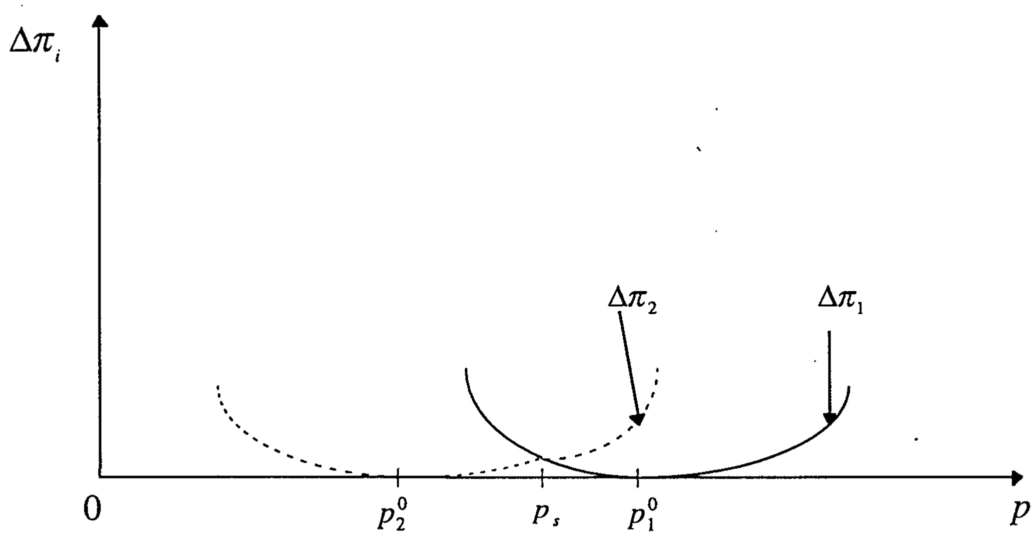


**FIGURE 6.** *The difference in profit between a market and non-market system.*





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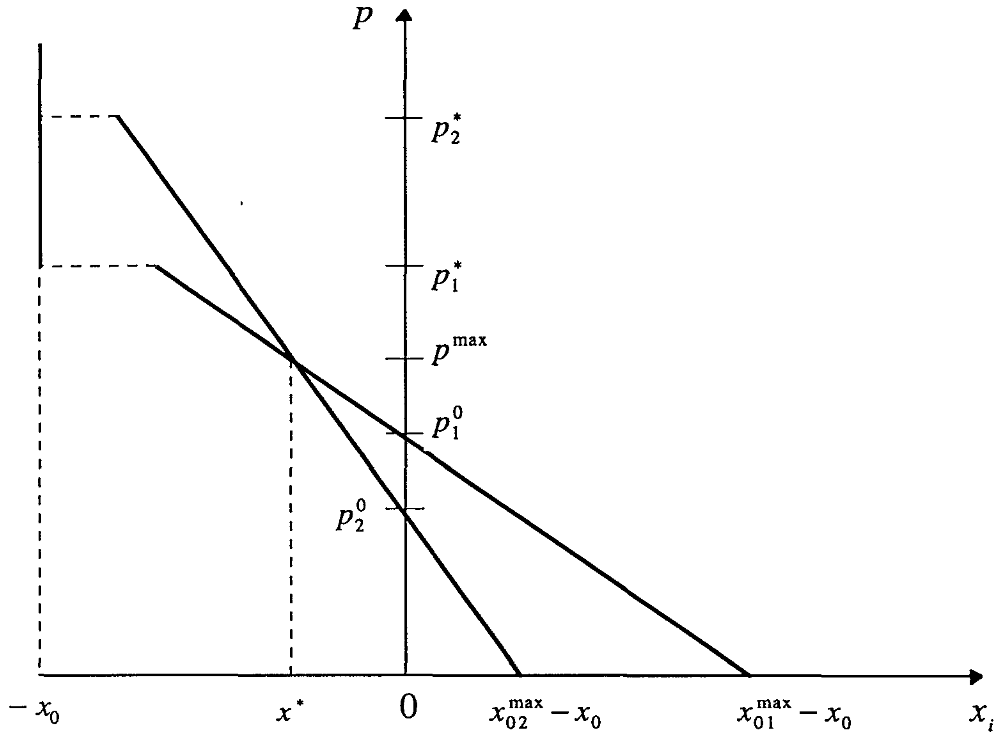


**FIGURE 7.** *Excess demand with modern and traditional technology.*

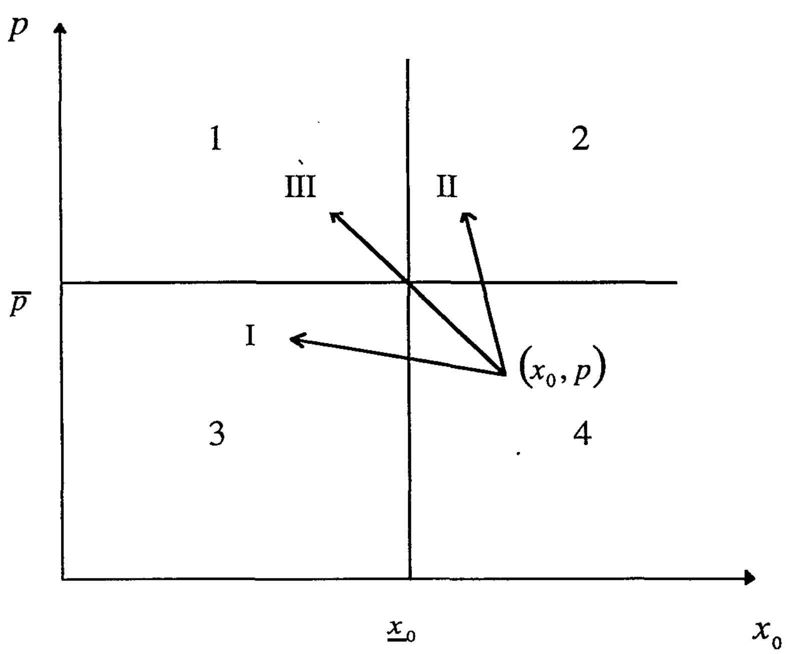


**FIGURE 8.** *Investment thresholds with a market versus non-market system.*

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